On the epistemological significance of Plato's theory of ideal numbers

By Samuel Scolnicov, Jerusalem

καὶ πάντα γα μὰν τὰ γιγνωσκόμενα ἀριθμὸν ἔχοντι· οὐ γὰρ
οἷόν τε οὐδὲν οὔτε νοηθῆμεν οὔτε γνωσθῆμεν ἄνευ τούτου.

Philolaus 44 B 4 DK

'This much, at least, I can say about all writers, past or future, who say they know the things to which I devote myself, whether by hearing the teaching of me or of others, or by their own discoveries – that according to my view it is not possible for them to have any real skill in the matter. There neither is nor ever will be a treatise of mine on the subject. For it does not admit of exposition like other branches of knowledge; but after much converse about the matter itself and a life lived together, suddenly a light, as it were, is kindled in one soul by a flame that leaps to it from another, and thereafter sustains itself. Yet this much I know – that if the things were written or put into words, it would be done best by me, and that if they were written badly, I should be the person most pained'[1].

Plato refers in the above passage to what is known as his 'Unwritten Teaching', the ἄγραφα, and these are, as we are here assured, the things περὶ ὧν σπουδάζει, the very core of his philosophy. We are allowed to catch a glimpse of it at best only at two and three removes – through references to, and fragmentary notes from, Plato's lecture 'On the Good' and his Theory of Ideal Numbers. This is the chief, and perhaps even the only, Platonic doctrine argued against by Aristotle in his Metaphysics and elsewhere.

The value of such evidence is widely disputed. Burnet, for example, would have us entirely dependent on Aristotle's testimony, and is supported in this view by Robin, Zeller, Stenzel, and lately by P. Wilpert[2]. As opposed to them, Shorey and Ritter dismiss Aristotle as an unreliable source of information[3]. Cherniss, at least

---

[1] Plato, *Letter* 7, 341 b 7–d 4 (Harward's translation).

[2] L. Robin, *La théorie platonicienne des Idées et des Nombres d'après Aristote* (Paris 1908; repr. Hildesheim 1963); J. Burnet, *Greek Philosophy* I (London 1928); E. Zeller, *Die Philosophie der Griechen* II 2 (1923); J. Stenzel, *Zahl und Gestalt bei Platon und Aristoteles* (3rd ed. Darmstadt 1959); P. Wilpert, *Zwei aristotelische Frühschriften über die Ideenlehre* (Regensburg 1949).

[3] C. Ritter, *Kerngedanken der platonischen Philosophie* (München 1931), Engl. tr. by A. Alles, *The Essence of Plato's Philosophy* (London 1933); P. Shorey, *Platonism, Ancient and Modern*

up to a few years ago, would push things to the extreme, and flatly deny the whole issue: Plato himself tells us in the Seventh Letter and in Phaedrus 274 e to 275b that there is no book of his on these matters and there will never be, because – so Cherniss explains – there never was such a Doctrine of Ideal Numbers. Aristotle's pronouncements on this subject contradict what is known to us from the dialogues and contradict one another. Plato never taught the identification of ideas and numbers, and the whole case is nothing but the consequence of Aristotle's misinterpretation[4].

A textual re-examination of these fragments is not within the purpose of this paper. It will be sufficient, I believe, to draw the reader's attention to de Vogel's paper in Mnemosyne[5]. De Vogel has not gone through the whole of Cherniss' argumentation, but she has succeeded, to my mind, in removing its sting, by carefully analysing some central passages. By now, one should, I think, agree with Wilpert[6] about the breakdown of the position of «the Aristotelian misunderstanding» (or even misrepresentation), especially as a result of the work of Stenzel and Jaeger. Even if their arguments may not be beyond assail, nevertheless it is no more possible to contend that Aristotle simply «did not understand» Plato.

The main interest of this paper lies with the Platonic side of the problem, rather than with the Aristotelian side of it. In the following pages I will try to show in what way the doctrine of ideal numbers is a logical development of the trend of thought displayed in the dialogues, and I shall tap the Aristotelian and post-Aristotelian sources mainly as an aid to what can be shown to exist already in the dialogues. The question of the reliability of these texts is indeed pertinent to our problem, but in order to keep this paper within reasonable boundaries, it seems advisable to refrain as much as possible from embarking on such a discussion. Moreover, the main line of argument in this paper does not depend necessarily on the Aristotelian evidence; on the contrary, in certain respects it could prove corroboratory to those texts[7].

But, even though the textual examination is indispensable, it is by no means sufficient. We must study Plato's philosophy in order to lay bare its major features, and seek in it the place of the doctrines that were handed down to us by

(Berkeley 1938). For a short summary of previous literature on the subject, cf. K. Gaiser, *Platons ungeschriebene Lehre* (Stuttgart 1963) 16–18.

[4] H. Cherniss, *Aristotle's Criticism of Plato and the Academy* (John Hopkins 1944; reissued New York 1962); *The Riddle of the Early Academy* (Berkeley 1945).

[5] C. J. de Vogel, *Problems concerning later Platonism*, Mnemosyne ser. 4, II (1949) 197–216. 299–318. See also H. J. Krämer, *Arete bei Plato und Aristoteles* (Heidelberg 1959) 380 ff.

[6] Op. cit. 124.

[7] Krämer und Gaiser would rather have the ἄγραφα δόγματα concomitant with the dialogues (Krämer 477 even puts them as early as the *Gorgias*). While I shall adduce in this paper some further considerations for seeing the unwritten doctrine implied already in the middle dialogues, I do not think there is any necessity in actually advancing the date of the lecture (or lectures) on the Good (as distinct from recognizing their *methodological* place in the sequence of Plato's works).

his disciples in his name. And should we be successful, these very doctrines would shed a new light on the corpus of the written philosophy. Evidently, this analytic-synthetic method is, in one important respect, circular, and it leaves ample place for preconceptions. Notwithstanding, within the close-knit fabric of Plato's thought, no single unequivocal thread can easily be found to lead us through it.

The chief preconception is perhaps the assumption of some kind of continuity in Plato's thought, thematic as well as chronological. Without this regulative principle, no meaningful interpretation is possible. Should someone point to philosophic or literary elements that would not fit into his general scheme of the Platonic philosophy, or should someone divide Plato's progress into so many stages, almost wholly disconnected but for biographic events[8] – his only argument would then be ab ignorantia.

This formal principle takes sometimes the form of presentation of some 'central thought' or 'essence' of Plato's philosophy, and subsequent interpretation of the text in accordance with it. This procedure is not wholly 'objectively' justifiable, but its justification is mainly 'functional'. I.e., these 'central thoughts' would, to a certain extent – but not entirely – be the crystallization of those aspects of Plato's philosophy in which the particular commentator happens to be interested. It is evident that a presentation of Plato's philosophy along these lines, even if it be in some respects unavoidable, cannot fail to be narrow and disproportioned. Plato himself, more than anyone else, has warned us against this danger. And the danger is felt only the more acutely when we come to tackle those things περὶ ὧν ἐσπούδαζε, and that, if it were possible to write them down, he himself would have done it better than anyone else.

Nevertheless, the development of Plato's philosophy, and perhaps that of the whole of Greek philosophy, display an inner logic and an inner dynamic, which focus our attention precisely on those matters. The problem of number – in other words, the problem of the one and the many – runs through the whole of Greek philosophy, from Thales up to Proclus, and even beyond him, and reached its most pointed expression in Plato.

I shall not be able to discuss the overwhelming wealth that flows from this problem as it is variously located by Plato in most different realms and contexts. I shall content myself with the discussion of one aspect of the problem which seems to me essential, namely the epistemological aspect. But – complying with the general character of the Platonic philosophy[9] – it will be impossible not to deal at the same time with its ontological aspect. Nonetheless, this discussion will center round the epistemological function of the ideas and of the numbers, and the justification for this choice of point-of-view must come from the conclusions

---

[8] E.g., G. Ryle, *Plato's Progress* (Cambridge 1966); cf. Y. G. Libes, *Plato's Progress – A review of Ryle's new book*, Iyyun 18 (1967) 22–44 (English summary, pp. 111–107).

[9] Cf. P. Haezrahi, *On the Perfect Being* (Jerusalem 1964) 160–161 (Hebrew); V. Brochard, *Etudes de philosophie ancienne et de philosophie moderne* (Paris 1954) ch. VIII–IX.

that it will yield. And if there is in this some disproportion – which is unavoidable – it should, at least, improve a little on previous views.

I

1. The 'critical' standpoint: the quest for the possibility of knowledge

The transition from the aporetic dialogues into the already 'Platonic' dialogues of the second period, which give us Plato's answers to Socrates' queries, can be clearly seen in the well-known passage from the Meno (80 d): καὶ τίνα τρόπον ζητήσεις, ὦ Σώκρατες, τοῦτο δ μὴ οἶσθα τὸ παράπαν ὅτι ἐστίν; ποῖον γὰρ ὧν οὐκ οἶσθα προθέμενος ζητήσεις; ἢ εἰ καὶ ὅτι μάλιστα ἐντύχοις αὐτῷ, πῶς εἴσῃ ὅτι τοῦτό ἐστιν δ σὺ οὐκ ᾔδησθα; 'And how will you enquire, Socrates, into that which you do not know? What will you put forth as the subject of enquiry? And if you find what you want, how will you ever know that this is the thing which you did not know?'[10]

This seemingly eristic contention casts a serious doubt upon the whole of Socrates' way of inquiry: the quest for the τί ἐστιν is impossible; there is no way out of Socrates' ignorance into real knowledge.

Plato's answer implies a definite break with the earlier dialogues. In effect, the solution is not advanced in Socrates' name, but in the name of «priests and priestesses wise in things holy». Plato does not refer directly to the question, and even refuses to answer it: 'And therefore we ought not to listen to this sophistical argument about the impossibility of enquiry: for it will make us idle, and is sweet only to the sluggard; but the other saying will make us active and inquisitive. In that confiding, I will gladly enquire with you into the nature of virtue (ἀρετὴ ὅτι ἐστίν)' (Meno 81 d 5–e 1; cf. 86 b 6–c 2).

To the same extent that the question is ἐριστικόν, the answer is dogmatic. Plato seems to be wholly conscious of this dogmatic character of his answer, and he does not attempt to justify it in any way. He takes for granted the moral superiority of inquiry over intellectual defeatism and asks for the conditions of the possibility of learning and knowledge[11].

Plato returns to this position at crucial points in the later dialogues, where the possibility of knowledge is explicitly under discussion:

'And yet, ... if a man, fixing his attention on these and the like difficulties, does away with ideas of things and will not admit that every individual thing has its own determinate idea which is always one and the same, he will have nothing on which his mind can rest; and so he will utterly destroy the power of reasoning, as you seem to me to have particularly noted' (Parm. 135 b–c).

---

[10] Jowett's translation. And so everywhere, unless otherwise stated.

[11] P. Natorp, *Platons Ideenlehre* (2nd ed. Leipzig 1923) 30ff.; J. Klein, *A Commentary on Plato's Meno* (Chapel Hill: Univ. of North Carolina Press 1965) 97 ff.

'And surely contend we must in every possible way against him who would annihilate knowledge and reason and mind, and yet ventures to speak confidently about anything' (Soph. 249 c 6–8).

Already in the first statement of the 'critical turn', in the centre of the Meno, three Platonic motives par excellence are present, which will serve as corner-stones in Plato's middle and later philosophy: the metaxy, the soul and the ideas. All these are grounded upon that confidence in the validity of knowledge, which validity Plato refuses to prove. These are minimum assumptions, without which no knowledge is possible.

Without ideas – this is Plato's contention in the Meno, in the Parmenides, in the Theaetetus, in the Sophist – knowledge is impossible. Only the idea, the καθ'αὑτό, can

one over many which unifies them, gives them meaning, binds them all together. The idea is, as Natorp puts it, «the unity of the conceptual content as against the plurality of what is conceived under it»[12]. The idea is the ἕν τι

192 b 9–c 1, Gorg. 468 b. 483 d, Meno 73 d); μία διὰ πάντων

μονοειδές

Republic it is spoken of the ἰδέα

ἐκ πολλῶν ... εἰς

εἰς

The idea is the unity, «it is forever the same and coming-into-being and perishing can never get hold of it» (Phileb. 15 a). In this sense Aristotle says of the ideas that they are the «essence of all other things» and the one is «the essence of the ideas» (Met. i 7, 988 b 4–6). The pure functions of thought, says Natorp, are in the whole nothing more than expressions of the one pure function of thought, namely, the One, whose different aspects they severally pick out. The One represents the idea in that it represents *one* idea[13].

Only by means of the synthetic unity of the idea is it possible to overcome the relativity of the world of the senses and ensure knowledge. From this point of view we could say with Natorp – although this statement is much too one-sided, as we shall see later on – that the ideas are «Erkenntnisfunktionen». The world of the senses is indeterminate («unbestimmt»), and there is not in it a thing that is one and equal to itself, and only on *such* an entity can knowledge be founded. Should knowledge be possible, this indeterminancy must come to an end (*peras*) in the synthetic unity of the idea[14]. The eighth hypothesis in the Parmenides shows us that without the One no science and no naming is possible: 'Then if one is not, there is no conception of any of the others either as one or many; for you cannot conceive the many without the one. – You cannot. – Then if one is not,

---

[12] Natorp, op. cit., Sachenregister s.v. Einheit u. Vielheit.
[13] Natorp, op. cit. 238. Cf. also Krämer 501 ff.: «Das Eins als Seinsprinzip», 137 ff.: «Sein nur insofern als Eins», and references there.
[14] Natorp, op. cit. 97.

the others neither are, nor can be conceived to be either one or many ? ... Then may we not sum up the argument in a word and say truly: If one is not, then nothing is? – Certainly' (166 b)[15].

The indeterminate, the boundless, the multiple as such, are inconceivable. If knowledge (and especially knowledge stricto sensu: scientific knowledge) is to be possible – and with Plato there is no doubt that knowledge *is* possible – then of necessity there is a being which is one, determinate and unchangeable[16].

But here we are bound to make a major reservation. The science Natorp speaks of is Kant's science: the science of the phenomenal world of the senses. According to Natorp's interpretation of Plato's theory of ideas, the veritable essence of the Platonic idea is in its relationship to the sensuous world and in its function of organization within this world. This conception of the idea is, at the least, one-sided. The idea should not be understood only as an epistemological function. The knowledge Plato speaks of in the Republic, in the Phaedrus and in the Symposium, and even in the Sophist, in the Politicus and in the Philebus, is the knowledge *of the world of the ideas*, and only derivatively, the knowledge of the world of the senses. True, the main point in Plato's arguments for the existence of ideas can perhaps be shown to be 'transcendental', but Plato takes his arguments as establishing the *existence* of ideas, and not only the possibility of their application within our knowledge[17]. The passage from the Parmenides quoted above, the references to the ideas throughout the middle dialogues, the Creation myth in the Timaeus, all these as well as many other passages point to the idea as fully existent and as responsible (αἰτία) for the world of the senses. The reduction of the ideas to their

---

[15] Cf. Haezrahi, op. cit. 205–206; Brochard, op. cit. 128; Natorp, op. cit. 270.

[16] Cf. Arist. *Met.* i 9, 990 b 12; Wilpert, op. cit. 31.

[17] It should be noted that according to the parables in the *Republic* vi, the constitutive force of the ideas in respect to the sensuous world is warranted precisely by that idea by equivocation, which is ἐπέκεινα τῆς οὐσίας (509 b), and therefore also ἐπέκεινα τοῦ νοῦ. In last analysis it is this idea that is responsible (αἰτία) for the being of the other ideas, and through them (but not only through them) for the being of the sensuous world. But multiplicity (in the world of ideas as well as in the world of the senses) does not stem from unity, but from a different source, as it will become apparent later on.

This positing of a real, transcendent (and not merely transcendental) source of being is not possible from the sole point of pure (scientific) knowledge. The positing of its reality – and with it the positing of the reality of the other ideas – is, therefore, from the point of view of the 'functionalistic' interpretation, a further step. Although Plato seems not to make any distinction between the 'functional' and the 'real' aspects of the idea, nevertheless, the fact that he describes the reality of the ideas by way of myth only shows that he was himself conscious of the impossibility of a formal proof concerning the reality of the ideas. Such is, e.g., the case in the parables in the *Republic*. On the other hand, in the 'dialectical' dialogues, the 'functional' aspects of the ideas is uppermost, as the neo-Kantian interpretation has shown – even if their contentions are somewhat exaggerated. The question regarding the (epistemological!) necessity of the reality of the ideas remains open, as this question cannot be answered, I think, within the framework of the neo-Kantian presentation of the problem. And indeed, even in the 'dialectical' dialogues, Plato does not give up his demand for the reality of the ideas. But he himself points out time and again that their reality is not derivable from their rationality (or not only from rationality alone).

mere function as transcendental categories, or even an exaggerated emphasis on it, would unduly shift the centre of gravity of the Platonic philosophy to the sensuous world, and would, at the same time, completely absorb the ideas into the soul. But Plato stresses time and again that the object of knowledge (and by this he means knowledge of the immutable being) is separate from knowledge itself, by which it is grasped[18]. The independence and the objectivity of the object of knowledge are to be stressed again and again[19]. If there is knowledge, there must be a *being* that is its object: «For the same thing is for knowledge and for being»[20]. Knowledge for Plato, as for Parmenides, cannot be but knowledge of being. The fact of knowledge implies as its condition the separate existence of the ideas. (cf. Rep. 479 a–480 a; Parm. 132 a–d; Tim. 51 d–52 d; Arist. Met. i 6, 987 a 32–b 10).

In Kant's terms – concludes Wilpert – the argument can be put as follows: The condition for scientific knowledge (with all due reservations about the differences in the concept of science) is the unity and the generality of its object. And as general and conceptually necessary knowledge is real («wirklich»), so is its object real too. This formulation brings forth also the difference between Kant and Plato: whereas to Kant the condition of possible knowledge comes back to the subject, to Plato it points forward to the object[21].

But Wilpert's formulation does not lay sufficient stress on the dual character of the idea: on the one hand it is a substance, existing καθ' αὐτό, but on the other hand it is, at the same time, an epistemological function ἐν ψυχαῖς. Only this duality can explain the fact of knowledge against the background of Plato's realistic outlook. Were the existence of the ideas merely functional, the whole of science would be defenceless in face of Gorgias' criticism, as Kant's solution in face of the scepticism of Solomon Maimon. The substantiality of the idea is, therefore, Plato's debt to «our father Parmenides», as his sole guarantee for the unconditioned veracity of our knowledge. And correlatively, the functionality of the idea is Plato's answer to Parmenides' uncompromising dichotomy and all that is implied by it. A concession on either of these aspects of the idea would mean despair from the possibility of valid science. The unity of the idea is not only a transcendental unity, but *also* a transcendent unity, existing in itself.

---

[18] Therefore, the direct vision of the ideas is indispensable in dialectic as a whole and in every step of it apart. The conceptual analysis is directed by reality and has no significance apart from it. Otherwise, dialectic would be no more than an empty game.

[19] Haezrahi, op. cit. 314. Cf. Plat. *Parm.* 132 b 3–10: Ἀλλά, φάναι, ὦ Παρμενίδη, τὸν Σωκράτη, μὴ τῶν εἰδῶν ἕκαστον ᾖ τούτων νόημα, καὶ οὐδαμοῦ αὐτῷ προσήκῃ ἐγγίγνεσθαι ἄλλοθι ἢ ἐν ψυχαῖς· οὕτω γὰρ ἂν ἕν γε ἕκαστον εἴη καὶ οὐκ ἂν ἔτι πάσχοι ἃ νυνδὴ ἐλέγετο. – Τί οὖν; φάναι, ἓν ἕκαστόν ἐστι τῶν νοημάτων, νόημα δὲ οὐδενός; – Ἀλλ' ἀδύνατον, εἰπεῖν.

[20] Parmenides 28 B 3 DK: τὸ γὰρ αὐτὸ νοεῖν ἐστίν τε καὶ εἶναι. On the translation, see W. K. C. Guthrie, *A History of Greek Philosophy* II (Cambridge 1965) 14. Cf. Wilpert, op. cit. 30–32.

[21] Cf. Wilpert, op. cit. 34–35.

2. The idea as unity and multiplicity

Parmenides stated the equation of rationality with unity. And Plato, para-doxically, pushed this thesis to its ultimate conclusion, namely, that simple and unrelated unity is utterly void and indistinguishable from not-being[22]. Pure unity is meaningless; its meaning can be given to it only by other unities clearly distin-guished from it. As the Sophist shows, being is not simply unity, but unity in a multiplicity, or, in Natorp's term, synthetic unity.

Here arises the problem: '... how each individual unity ... can be conceived either as dispersed and multiplied in the infinity of the world of generation, or as still entire and yet divided from itself?' (Phil. 15 a). And the paradoxical solution: ἐξ ἑνὸς μὲν καὶ πολλῶν ὄντων τῶν ἀεὶ λεγομένων εἶναι, πέρας δὲ καὶ ἀπειρίαν ἐν αὐτοῖς σύμφυτον ἐχόντων. '... whatever things are said to be are composed of one and many, and have the finite and infinite implanted in them' (ibid. 16 c).

Being is always conjoined with unity and unity with being. But every being is not only a simple unity, but also a multiplicity. And this is the case not only in respect of the relation between the idea and its sensuous instantiations, but the problem, as we shall see, exists also in the realm of the ideas. As the Sophist shows, there is not reality without distinctions and without relations, and unity itself, for the sake of which every distinction and every relation is to be suppressed, unity itself cannot exist without being, at the same time, plurality. Unity and plurality exist together in everything.

The same being, one and identical with itself, e.g. the sophist, can be diversely predicated, and is, therefore, a multiplicity of beings: he is a hired hunter of rich young men, a broker and retailer of knowledge, he is an athlete exercising in eristic, he is a purifier of the soul of what hinders it from attaining knowledge and he is a juggler, all in one. The idea of the sophist partakes in all these other ideas. In other words, the idea of the sophist forms a new *unity* out of the multi-plicity of these ideas, and binds them together with a bond that is not one of identification, but a predicative, i.e. synthetic bond[23].

Every idea contains, therefore, within itself, a multiplicity, and this multiplicity *is given* in it, and is discovered by the dialectical analysis: δεῖν οὖν ἡμᾶς τούτων οὕτω διακεκοσμημένων ἀεὶ μίαν ἰδέαν περὶ παντὸς ἑκάστοτε θεμένους ζητεῖν – εὑρή-σειν γὰρ ἐνοῦσαν – ἐὰν οὖν μεταλάβωμεν, μετὰ μίαν δύο, εἴ πως εἰσί, σκοπεῖν, εἰ δὲ μή, τρεῖς ἤ τινα ἄλλον ἀριθμόν, καὶ τῶν ἐν ἐκείνων ἕκαστον πάλιν ὡσαύτως, μέχρι-περ ἂν τὸ κατ᾿ἀρχὰς ἓν μὴ ὅτι ἓν καὶ πολλὰ καὶ ἄπειρά ἐστι μόνον ἴδῃ τις, ἀλλὰ καὶ ὁπόσα. '... seeing, then, that such is the order of the world, we too ought in every enquiry to begin by laying down one idea of that which is the subject of enquiry; this unity we shall find in everything. Having found it, we may next proceed to look for two, if there be two, or, if not, then for three or some other

[22] *Parm.* 141 e 10; cf. Ritter, op. cit. 164 ff.
[23] Brochard, op. cit. 133. 140.

number, subdividing each of these units, until at last the unity with which we began is seen not only to be one and many and infinite, but also a definite number' (Phil. 16 c–d).

The synthetic character of the idea is *given: τούτων οὕτω διακεκοσμημένων*. The predicative bond is unexplained; it was handed down to us by the gods (Phil. 16e: *οἱ μὲν οὖν θεοὶ ... ἡμῖν παρέδοσαν*). The role of the dialectic is to unfurl before us the synthetic fabric of the world of the ideas. And as we shall see below, it is this synthetic character of the idea which gives it meaning, for the bare One is inexpressable (Parm. 137 c. 142 b; Soph. 244 e–245 a)[24].

The absolute unity is not susceptible of being known[25]. If the ideas are many and are simple unities and are different from one another, the difference between them is irrational, as Melissus showed[26]. The 'Sophist' brings us paradoxically to the conclusion that this irrationality can only be overcome by letting the plurality into the idea itself. The decomposition of the unity of the idea into a plurality of partial determinations gradually dissolves the irrationality of the idea[27].

But this throws the door open for the plurality to enter the world of ideas itself. The idea is no more simply 'the one over the many', but it contains in itself a multiplicity, in so far as it partakes – as *idea* – no less than sensuous particulars, in other ideas. Plurality, the sign par excellence of irrationality, is essential to the idea no less than unity.

---

[24] Brochard, ibid. 127. 136.

[25] *Parm.* 142 a 4–6, *οὐδ' ὀνομάζεται ἄρα οὐδὲ λέγεται οὐδὲ δοξάζεται οὐδὲ γιγνώσκεται, οὐδέ τι τῶν ὄντων αὐτοῦ αἰσθάνεται*. Cf. on not-being, *Soph.* 238 c 10. Cf. Parmenides 28 B 8, 1. 15 DK.

[26] 30 B 8 DK.

[27] Cf. Ritter, op. cit. 209: «But even these ideas contain something irrational, viz., their qualitative character which must be unreservedly accepted. This is the irrationality which the *Philebus* has in mind, viz., the permanent characteristics of all Being, which, by a careful logical consideration, can be classified under the general concept and which may thus be divested of their irrationality more and more, but there is something in them which always resists rational explanation.»

The problem whether this procedure is finite is certainly not an easy one. I shall content myself with adducing some arguments in favour of this view. If the diairetic procedure is finite, it comes to an end in the *ἄτομα εἴδη*, even if we concede that the components of their definitions (viz., the *μέγιστα γένη* or the 'one' and the 'indefinite duality') are irrational insofar as they are unanalysable (cf. above note 21). And, on the other hand, an infinite number of steps would destroy the numerical structure of the idea. Therefore, one should perhaps agree with Ritter in that «there is something in them which always resists rational explanation». But this does not mean an infinite procedure, as Ritter's words could be taken to imply. An infinite procedure can be rational (or can be rationalized) only within a framework of a theory of infinitesimals, such as Leibniz' or Solomon Maimon's. Whereas the rationality of the idea is the rationality of the fixed structure, of the well-defined relations between the qualitative moments in its definition. Nevertheless, these moments themselves – insofar as they are qualitative moments in the definition of this particular idea, and disregarding the possibility of their being in their turn analysed into *their* respective moments, up to the ultimate components of every idea – these moments as such are irrational.

There could be, perhaps, some interest in noting that Marsilius Ficinus too interpreted the Platonic process of knowledge as implying an infinite progress. Cf. Marsilius Ficinus, *Theologia Platonica de immortalitate animorum* (1482) lib. VIII, cap. 16.

Plato asked himself whether the ideas really are the ultimate explanation, the arch-principle of Being. The Republic answered this question in the negative, in respect of their *being*. The later dialogues, by way of elucidating the problem of the one and the many *within the world of the ideas*, made clear that their *unity*, i.e. their rationality, is not readily understandable. The problematic relationship of the one and the many makes now its appearance even before the participation of the sensuous particulars in the ideas. Although unity is rationality, nevertheless, without plurality the world of ideas is not understandable. If the world of ideas is to be a cosmos, it must be a multitude. And so we are back to the problem of participation, but this time participation of ideas in one another. The irrationality of the world of the senses now threatens the world of the ideas itself[28].

But the sequel of the passage in the Philebus 16 d warns us not to hurry too much in the transition to the sensuous world: we should not pass at once (εὐθύς) from the one to the indefinite, but we should stop at «what is between» (τὰ μέσα). So also in the example of the letters (ibid. 17 a 8 ff.): the voice is both one and indefinite (ἄπειρον). But the knowledge of the letters is not exhausted in this simple statement, that the voice is both one and many (ἄπειρον). On the contrary, it consists exactly in the further specification of this statement, viz. in saying *how many* voices are they and *of what types* (πόσα τ' ἐστὶ καὶ ὁποῖα).

The ideas cannot be the first explanation, because they are themselves composed out of elements. After the question about the elementariness of the ideas is answered negatively, the later dialogues raise the question of the elements of knowledge and being, which will be in the centre of Plato's thought from this period on. For the principal task of the philosopher is an analytical one: he must lay bare the ultimate elements of Being. Like he who studies the speech, he must come down to the letters (voices) of which speech is composed. But further than these elementary voices he cannot go[29].

---

[28] Wilpert, op. cit. 143–144. But this danger is not without compensation. From now on, all the problems referring to Being are summarized in a single one: the problem of the relations of the ideas among themselves. Cf. Brochard, op. cit. 149: «Tous les problèmes relatifs à l'être se réduisent à un seul, qui est: le rapport des Idées entre elles et surtout des Idées les plus hautes, celles auxquelles participe tout ce qui existe, en dehors desquelles rien ne peut exister ni être conçu.» Insofar as rational inquiry is necessarily held in the medium of the ideas, it seems to me that Brochard is right. But the *Timaeus* makes the point that the 'non-ideal' world cannot be approached by dialectic, but only by myth. Whereas the *Sophist*, the *Politicus* and the *Philebus* clarify the question of the relations of the ideas among themselves, the *Timaeus* comes back to the problem of the relations among the ideas, the souls and the sensuous world. Viewed from this aspect, the *Timaeus*, which stresses the position sui generis of the sensuous world, is off the main path of development of Plato's later dialectic as it is exposed, e.g., in the *Sophist* and the *Philebus*.

[29] *Polit.* 278 a–d. 285 d; *Crat.* 424 e; *Theaet.* 201 e; *Tim.* 48 b; *Soph.* 252 b; Arist. *Met.* i 2, 982 b 9; *Protr.* fr. 52 ll. 2–4 Rose; Sext. Emp. *Adv. math.* 10, 250. Cf. Stenzel, op. cit. 13–18. 154–156; Wilpert, op. cit. 129.

3. Dialectic: the science of structure

The science that exposes the rationality of the world of ideas is dialectic, which is already presented in the Phaedrus as the science of bringing together and separating of ideas, of synthesis and analysis (synagoge kai diairesis). And more explicitly in the Sophist 253 d: [ὁ διαλεκτικὸς] μίαν ἰδέαν διὰ πολλῶν, ἑνὸς ἑκάστου κειμένου χωρίς, πάντη διατεταμένην ἱκανῶς διαισθάνεται, καὶ πολλὰς ἑτέρας ἀλλή-λων ὑπὸ μιᾶς ἔξωθεν περιεχομένας, καὶ μίαν αὖ δι' ὅλων πολλῶν ἐν ἑνὶ συνημμένην, καὶ πολλὰς χωρὶς πάντη διωρισμένας. '(The dialectician) discerns clearly one Form everywhere extended throughout many, where each one lies apart, and many Forms, different from one another, embraced from without by one Form; and again one Form connected in a unity through many wholes, and many Forms, entirely marked off apart'[30].

The image of the diairesis and the image of the symploke are one and the same: the division of the ideas and the determination of the place of their components are, at the same time, also the decomposition of each of the components of the 'higher' idea and the exposition of the inner structure of each of them. The definition of angling by means of diairesis not only determines the 'place' of the idea of angling within the diairetical picture of the arts, but gives us as well the inner structure of the idea of angling as «the acquisitive art of hunting water animals by striking by day with a barb from below upwards». The logical method of definition is inseparable from the objective content of the idea. The steps of the definition are not merely auxiliary in determining the content of the idea, but they are constituent elements of the idea itself[31].

The structure of the idea is the structure of the cosmos of the ideas, and the meaning (content) of the particular idea is nothing but the unfolding of its relations with the other ideas, each of which is a unity in itself, different from each other and entirely marked off apart. If the idea is to be rational, it must be expressible in a discursive decomposition, it must have a distinctive structure which marks it off from other ideas and relates it to these other ideas.

The method of discursive decomposition of the synthetic unity of the idea is the dialectic. The considerations of the Sophist, preceded by those of the Republic, led Plato to a hierarchic view of the world of ideas, as it is expressed, e.g., in the diairetical picture, or in the passage quoted above. The idea cannot be thought of in itself, it must be linked to other ideas, to which it is subordinated, by which it is embraced and shot through. Without these 'higher' ideas, no 'lower' idea is thinkable, i.e., according to the Parmenidean-Platonic conception, it cannot exist. So, e.g., one cannot think the concept 'man' without the concept 'animal', and one cannot conceive the physical body without conceiving the geometrical body,

---

[30] Cornford's translation. Cf. Natorp, op. cit. 286–287; F. M. Cornford, *Plato's Theory o Knowledge* (London 1935; repr. 1966) 263; Haezrahi, op. cit. 277–278.

[31] Cf. Arist. *An. post.* ii 13; *Met.* vii 12; viii 6.

nor the body without the plane, nor the plane without the line, nor the line without the point[32]. Here it is not the case of derivation of the line from the point, the plane from the line, and so on, but of maintaining that the line is a *condition* for the plane, the point a *condition* for the line, and in this sense they are *prior* to them, or, in still other terms, that they are principles (ἀρχαί) or elements («letters», στοιχεῖα). Man *participates* in the animal, the body in the plane, etc. The idea in its isolation is not thought and has no being without its conditions above it. Of necessity, all the ideas are conditioned by the highest ideas[33].

In the Sophist Plato brought the pyramid of the ideas down to the ἄτομα εἴδη as the lowest limit of rationality, below which there is only the apeiria of the sensuous multiplicity. But the diairetic decomposition itself has shown us that the problem of the one and the many does not arise specifically at the meeting of the ἄτομα εἴδη with the world of the senses, but the problem is there already in the world of the ideas and in every idea as such[34].

The necessity of introducing multiplicity into the idea is well seen in the question of the 'matter of the ideas', that Plato is reported to have taught, the principle of differentiation in the ideal world, as the 'indeterminate duality' (ἀόριστος δυάς).

4. Prior and posterior

The relationship between the physical and the geometrical bodies between the latter and the plane, between the plane and the line, and between the line and the point, deserves a more careful examination. Wilpert[35] draws our attention to Arist. Protrepticus, fr. 52 Rose (= fr. 5 Ross): Αἴτιά τε μᾶλλον τὰ πρότερα τῶν ὑστέρων· ἐκείνων γὰρ ἀναιρουμένων ἀναιρεῖται τὰ τὴν οὐσίαν ἐξ ἐκείνων ἔχοντα, μήκη μὲν ἀριθμῶν, ἐπίπεδα δὲ μηκῶν, στερεὰ δὲ ἐπιπέδων.

This is exactly the relationship between the 'syllables' and the 'letters'. The passage above speaks of things related as to their being. Of this kind is the relation

---

[32] Cf. Wilpert, op. cit. 151–152; Stenzel, op. cit. 110–114.

[33] Cf. Wilpert, op. cit. 103. But this series of conditions and conditioned is not dependent on the One alone. For Plato multiplicity (and in particular the multiplicity of the ideas) is not derivable from the bare One. Wilpert (p. 179) tries to effect such a derivation: «Menge ... ist selbst wieder nicht denkbar ohne Bezug auf letzte Elemente, deren Vielzahl sie ist. So enthält die Zahl auch ihrem Inhalt nach die Beziehung auf das Eine. Aber diesmal ist es ein Eines, dessen Gegensatz nicht das andere ist, dem vielmehr die Menge, das Viel gegenübersteht. Besser würde man in diesem Fall überhaupt nicht von einem Gegensatz reden, denn das Eine als Element ist selbst nicht ohne Relation zur Menge, deren Element es ist. Die Vielheit setzt sich zusammen aus elementaren Einheiten.» But this conception of number is not the Platonic, in which the elementary multiplicity is apeiron. For Plato, the dualism between relation and substance is irreducible; the 'ontological leap' does not displace the ideas as conditions of the world. Cf. further, below p. 96 s. and n. 75.

[34] Cf. Arist. *Met.* i 6, 987 b 20ff.; Simpl. *In Phys.* 151, 6–9 Diels; 247, 33–248, 20. Stenzel, op. cit. 64; O. Toeplitz, *Das Verhältnis von Mathematik und Ideenlehre bei Plato*, Quellen u. Stud. z. Gesch. d. Math. 1 (1929) 20; A. E. Taylor, *Form and Number: a study in Plato's metaphysics*, Mind 36 (1927) 421.

[35] Op. cit. 148ff.

of the body to the plane and the planes to the lines. But it is clear that the body is not 'composed' of planes, or the plane of lines, in a mathematical sense. That Plato did not have in mind a geometrical construction of lines out of points and so on, is, to my mind, apparent from the theory of 'indivisible lines' ascribed to him[36]. Nevertheless, the line is, in a particular sense, a presupposition of the plane, and the plane, a presupposition of the body, etc. The two-dimensional extension is not to be thought without the one-dimensional extension, not because of some psychological or transcendental human shortcoming, but because of the nature of the object itself. In line with the Parmenidean-Platonic primary presupposition: the two-dimensional extension cannot *be* without implying the existence of the one-dimensional extension[37].

The same holds good for the genus-species relation. One cannot think of man without thinking at the same time of animal or of body. Here again, the impossibility is grounded on the nature of the object. If man is not animal, he is not man. If one suspends (ἀναιρεῖ) the essence 'animal', one suspends with it the essence 'man', which τὴν οὐσίαν ἐξ ἐκείνου ἔχει. But not conversely. One can think of 'animal' without thinking at the same time its determination 'man' or 'beast'. What one understands by 'animal' is not dependent upon its specifications. This logico-ontological relation is characterised by Plato as affording the suspension of the 'lower' idea without implying the suspension of the higher, but not conversely. Or, in other terms, there is among the ideas an order of prior and posterior. As Aristotle tells us clearly: τὰ μὲν δὴ οὕτω λέγεται πρότερα καὶ ὕστερα, τὰ δὲ κατὰ φύσιν καὶ οὐσίαν, ὅσα ἐνδέχεται εἶναι ἄνευ ἄλλων, ἐκεῖνα δὲ ἄνευ ἐκείνων μή· ᾗ διαιρέσει ἐχρήσατο Πλάτων. 'Some things then are called prior and posterior in this sense, others in respect of nature and substance, i.e., those which can be without other things, while the others cannot be without *them* – a distinction which Plato used'[38].

Exactly in this logico-ontologic sense it is said of the line that it is 'prior' to the plane and of the plane that it is 'prior' to the body. This is also the relation of the geometrical body to the physical. The same line of thought that leads from the line to the plane leads also from the mathematical body to the physical body. In both cases, the first is prior to the second in that it is the logico-ontologic condition of the other. Stenzel would have here a leap from the immaterial to the material, from mathematics to physics, as in the Pythagorean account of the

---

[36] Cf. *Met.* i 9, 992 a 22. Burnet, *Greek Philosophy* (London 1914) 262. And cf. further the Peripatetic treatise *De lineis insecabilibus*.

[37] But I cannot agree with Gaiser that the structure of space provided Plato with the model for his ontological hierarchy. Gaiser's arguments are mainly based on the *Timaeus*, which seems to me, as the passage from the *Protrepticus* quoted above, to deal with a special case of the principle of ontological hierarchy. It is conceivable, although not at all necessary or even very probable, that Plato took his cue from the structure of space, but there is little justification for carrying over to other hierarchies features that are specifically or primarily spatial.

[38] *Met.* v 11, 1019 a 1 (ἐχρήσατο Ross).

genesis of the world. But as Wilpert points out, rightly, I think, according to the foregoing considerations, the difference between physics and mathematics (as far as materiality is concerned) is irrelevant. Plato asks for the logico-ontologic conditions of the world, and within this frame geometry is the condition of physics. This can be clearly seen in the description of the sciences in the Republic VII[39].

5. *The idea between the one and the infinite*

The process of diairetic decomposition of the idea is not infinite[40]. The rationality of the idea has its clear expression in the fact that its components can be hierarchically ordered, and the 'transparency' of the idea is due to its structure as it appears to us in the diairetical picture. If the diairetical process were infinite, the idea could not attain rational expression in thought and language, and would differ in nothing from the apeiria of the sensuous world.

The idea holds a middle place between the two irrational extremes. The absolute One is ineffable and unthinkable, it is closed within itself, and it is beyond rationality, even though it is the fountainhead of all rationality. Neither is absolute multiplicity susceptible of expression, in so far as every expression implies unity. Between the One, which is alogon, and the infinite, which is likewise not to be grasped, stands *number*: [δεῖ ἡμᾶς] ζητεῖν ... τὸν ἀριθμὸν ... πάντα τὸν μεταξὺ τοῦ ἀπείρου τε καὶ τοῦ ἑνός (Phil. 16 d–e).

The idea is multiple, but it is not absolute multiplicity. It is determined multiplicity, multiplicity in unity. The transition from the one to the infinite is not immediate. The diairesis progresses by gradually accumulative determination, until a stage is reached in which no new determination is possible or desired. And from this last determination on, we are in the domain of the apeiria of the sensuous world[41].

But before the idea dissipates into the infinity of the sensuous world, it is *determined* by diairetical steps, which are numerically fixed: πῶς ἔστιν ἓν καὶ πολλὰ αὐτῶν ἑκάτερον, καὶ πῶς μὴ ἄπειρα εὐθύς, ἀλλὰ τίνα ποτὲ ἀριθμὸν ἑκάτερον ἔμπροσθεν κέκτηται τοῦ ἄπειρα αὐτῶν ἕκαστα γεγονέναι; '... how they are one and also many, and are not at once infinite, and what number is to be assigned to either of them before they pass into infinity' (Phil. 18 e–19 a).

The determinateness of the idea, in distinction from the indeterminateness of the sensuous world, its rational structure that can be exactly expressed, is given through the numerical essence of the idea. This numerical essence is expressed in the well-measured succession of diairetic steps, which fixes the place of the particular idea within a network of hierarchical relationships with other ideas[42].

---

[39] K. H. Ilting, *Platons 'Ungeschriebene Lehren': der Vortrag 'Über das Gute'*, Phronesis 19 (1968) 1–31, is therefore right in concluding that there is here no case for a «Versuch einer Deduktion der Welt» (p. 30).

[40] See n. 27 above.

[41] Cf. Natorp, op. cit. 301.

[42] Cf. *Phil.* 16 c 5ff. On the primary importance of the concept of *order* and *measure* in this

Plato asked himself whether the ideas were indeed the ultimate elements of Being. Now, by way of groping with the problem of the One and the many he came to the conclusion that the rationality of the ideas is in no way readily understandable. The transition to the sensuous instantiations of the idea is not the first occurrence of the dissolution of the one into the many. Already the world of ideas itself and every idea within it are each an articulated building (or, better still, animal), and there is in every member of it a breaking-down of the one into many and conversely a participation of every multiplicity in a higher unity. But this amounts to saying that the ideas are not elements, in that they are not simple.

The articulated cosmos of the ideas, by way of participation of every idea in other ideas: such is the solution to the problem of the one and the many offered us by Plato in the later dialogues. Furthermore, these dialogues show us that the cosmos of ideas is ordered according to measure. The specification of the higher idea into the lower ideas is not arbitrary and the transition from the one to the infinite is not immediate[43].

The participation of ideas in each other is ordered by numerical («zahlmässige») relations and expressed by them. The order of the ideas is determined by the numerical nature of the system of ideas (and, correlatively, by the numerical nature of the idea itself), namely, by the 'condition' and the 'conditioned', the 'prior' and the 'posterior'. Between the One and the indeterminate infinity stands number as 'a determined multiplicity'[44].

Every idea is characterized by its 'place' in the cosmos of ideas, and this place is expressed by the contents it holds as its partial determinations (i.e. the other ideas it participates in), and by the structure of this contents, which is determined and ordered univocally according to the prior and the posterior. This determinateness lifts up from the idea its opaqueness and relates it to higher ideas. It is this articulation of the multiplicity in the one which is the guarantee of the rationality of the idea, rationality that was doubted because of multiplicity as such. The idea, in that it is a determined multiplicity, participates in number, or, as this position was also formulated, ideas are numbers.

II

1. The relation between ideas and numbers (I)

Ideas participate in numbers or, in another formulation, ideas are numbers. Is this really merely a question of formulation? He who says that ideas are num-

---

context, cf. Krämer, op. cit., esp. 144ff.: «Ordnung als Seinsstruktur», and 324–325: «Ordnung ist die Seinsstruktur schlechthin». See also J. Klein, *Greek Mathematical Thought and the Origin of Algebra*, tr. E. Brann (Cambridge, Mass. 1968) 89ff.

[43] Cf. Wilpert, op. cit. 143–145.

[44] Arist. *Met.* v 12, 1020 a 13: πλῆθος πεπερασμένον. Cf. below p. 92.

bers does he mean no more than that ideas 'partake' in numbers; or does he, perhaps, contend in all seriousness that ideas *are* numbers? Robin[45] can be taken as a representative of the first opinion. As he would have it, ideas are not numbers in the sense that they are identical to numbers, but they are numbers in the same sense that man is animal. I.e., in his view, the relation is one of subordination of one concept to another.

As against him, Ross, in his Introduction to Aristotle's Metaphysics, is peremptory: «Aristotle implies quite definitely that Plato held all the Ideas to be numbers»[46]. In their commentary to the Metaphysics of Theophrastus Ross and Fobes are more reserved, perhaps because the passage in question squares better with Robin's interpretation than with their own: Πλάτων μὲν οὖν ἐν τῷ ἀνάγειν εἰς τὰς ἀρχὰς δόξειεν ἂν ἅπτεσθαι τῶν ἄλλων εἰς τὰς ἰδέας ἀνάπτων, ταύτας δ'εἰς τοὺς ἀριθμούς, ἐκ δὲ τούτων εἰς τὰς ἀρχάς, κτλ. 'Now Plato in reducing things to the ruling principles might seem to be treating of the other things in linking them up with the Ideas, and these with the numbers, and in proceeding from the numbers to the ruling principles, etc.'[47]

Ross' and Fobes' commentary to this passage is worth while quoting at length, as it may serve as a good summary of the contention: «This [sc. ταύτας δ'εἰς τοὺς ἀριθμούς], if taken strictly would mean that the numbers occupied, for Plato, a higher grade in the hierarchy of being than the Ideas. The numbers referred to cannot, of course, be the mathematical numbers, which were «intermediate» between the Ideas and the sensible things. They must be the ideal numbers, i.e. the essence of the integers; and these are themselves Ideas. Thus the theory would be that the Idea-numbers form a superior class from which all other Ideas are derived. Against this we have to set Aristotle's repeated statement that in the Platonic theory the Ideas (i.e. all the Ideas) were numbers; cf. Met. 919 b 9: εἴπερ εἰσὶν ἀριθμοὶ τὰ εἴδη, 992 b 15 ταῦτα γὰρ οὔτε εἴδη οἷόν τε εἶναι (οὐ γὰρ εἰσὶν ἀριθμοί), 1073 a 18 ἀριθμοὺς γὰρ λέγουσι τὰς ἰδέας οἱ λέγοντες ἰδέας, 1083 a 17 εἴπερ εἰσὶν ἀριθμοὶ αἱ ἰδέαι, 1084 a 7 εἰ πᾶσα ἰδέα τινός, οἱ δὲ ἀριθμοὶ ἰδέαι.

The present passage is the main evidence for M. Robin's view that the Numbers were superior to the Ideas, and related to them as mathematical numbers were to sensible things. But we can hardly accept T.'s testimony against that of Aristotle, from whom he probably derived his knowledge of Plato's ἄγραφα δόγματα. T.'s testimony cannot be ignored, however, and it seems possible to reconcile his statement with those of Aristotle. Plato may be supposed to have reached his view in some such way as this: Reflecting on the nature of the straight line, he would observe that it is completely defined by two points in space, in the sense that through two given points one and only one straight line can pass. He therefore described 2 as

---

[45] Op. cit. 454–461. For a good summary of the different stands on this problem see Wilpert, op. cit. 160ff.

[46] W. D. Ross, Introduction to *Aristotle's Metaphysics*, vol. I (Oxford 1924) lxvii.

[47] Theophr. *Met.* 6 b 11–14 Ross-Fobes.

the Form of the straight line. Similarly 3 was the Form of the plane. And since the simplest rectilinear solid, the tetrahedron, is completely determined, if we can give its four courner-points, 4 was the Form of the solid. From this he seems to have reached the general view that for each entity there is some number which states its structure and nature in the most abstract possible way; thus each Form was said to be a number. But the same number might be the Form of more than one thing. E.g., 4 was the Form of justice as well as of the solid, since justice involves two persons, and two 'honours or possessions' to be divided between them. The solid and justice could therefore be regarded as the number 4 manifested in two different materials, and a higher point of abstraction was reached when one spoke of 'the number 4' than when one spoke of 'the Form of the solid' or of 'the Form of justice'.

Thus Aristotle is justified in characterizing the theory by saying that the Platonists described the Forms as numbers, rather than by saying that they described the numbers as Forms; and Theophrastus is justified in saying that they linked the Forms up with numbers as with something superior to them.»[48]

The argument seems to have several weak points:

1. I do not think that the passages from the Metaphysics that were brought in to prove the identification of ideas and numbers are sufficiently clear and unambiguous so as to support such a conclusion. They can, at least, be interpreted both ways; and a scholar of the stature of Robin did not find them compelling.

2. Ross and Fobes do not think it necessary for their purposes to try and distinguish, where it is possible, between Plato and the Platonists. Their paragraph opens with Plato's agrapha and closes with a résumé of the theory of Ideas-Numbers of the Platonic School. The distinction is indeed difficult, and not always feasible or necessary, but to our point it is most important. Among the passages brought in by Ross and Fobes, the only one that refers beyond doubt to Plato himself is Met. 1073 a 18. But this passage offers some serious difficulties to the interpreter, as Ross himself notes elsewhere[49].

3. Likewise, the places adduced by Ross in his Introduction to Aristotle's Metaphysics[50] in favour of the identification of ideas and numbers by Plato do not

[48] W. D. Ross and F. H. Fobes, *Theophrastus: Metaphysics*, with tr., comm. and intr. (Oxford 1929; repr. Hildesheim 1967) 58 (ad 6 b 13).

[49] The passage runs: περὶ δὲ τῶν ἀριθμῶν ὁτὲ μὲν ὡς περὶ ἀπείρων λέγουσι ὁτὲ δὲ ὡς μέχρι τῆς δεκάδος ὡρισμένων. Cf. *Phys.* iii 6, 206 b 32: μέχρι γὰρ δεκάδος ποιεῖ [sc. Πλάτων] τὸν ἀριθμόν. Ross ad loc. in his ed. of the *Physics* (Oxford 1936) 557: «Aristotle ascribes the view to some of the Platonists in *Met.* 1073 a 20, 1084 a 12, 31. Plato may have thought that the numbers higher than 10 could be treated as mere combinations of the numbers up to 10; though this involves treating the higher numbers, contrary to his own principle, as συμβλητοί. But it is quite possible that Aristotle is taking seriously some mere *obiter dictum* of his master.» For another view, see Gaiser 124ff.

[50] Op. cit. (above n. 46) lxvii: i 9, 991 b 9. 21; xiii 8, 1084 a 12–25; xiv 5, 1092 b 8. 14. 16–23.

carry conviction. Some of these places plainly refer to the Platonists and not to Plato himself. De anima 404 b 18ff. is something of a problem[51].

4. But the main difficulty lies in Ross' and Fobes' interpretation of the relation between the numbers and the line, the plane and the body. If the account given by Stenzel and Wilpert of the relation of prior and posterior is correct, then number, precisely by being the 'form' of the line, the plane and the solid, *is the condition* of the line, the plane or the solid. The process that Ross calls 'abstraction' is the ascent from the conditioned to its condition, as this ascent is described in the parable of the divided line. If this is so, number (in general or a particular number) is prior – in the sense of priority in the diairetical picture – to the line, the plane or the body. At any rate, Ross seems to be right in maintaining that the passage speaks of ideal and not of mathematical numbers.

2. The relation between ideas and numbers (II)

Simplicius too, in rendering Alexander Aphrodisias' notes on Plato's lecture 'On the Good'[52], states that what is said of the ideas is said of number, but not conversely. So Aristotle too, in his dialogue 'On Philosophy': There is another kind of number, different from the mathematical. Ideal numbers, says Aristotle further, are related between themselves as prior and posterior[53].

But perhaps all this does not amount to a proof of the claims on either side. As Wilpert points out, one cannot lightly dismiss Theophrastus' or Alexander's evidence; on the other hand, it is impossible to overlook Aristotle's own sayings. And, at any rate, all the evidence we can rely on stems from one single source: Aristotle. When textual evidence can be no more of any help, only the way of material considerations remains open to us: i.e., to point out the reasons that could incline Plato to one side or the other. True, every decision on such grounds in favour of this or that interpretation would be in danger of slipping into a petitio principii: it must be grounded on the reasons that were supposed to lead Plato's thought, and these reasons cannot be inferred but from the same texts that are being interpreted[54].

Wilpert tries to escape between the horns of the dilemma. Following to some extent Ross and Fobes in their commentary to Theophrastus, Wilpert suggests that we accept both possibilities: As synthetic unit the idea is number and as

---

[51] Cherniss, *Criticism*, App. XI, claims that this passage refers to Aristotle himself and not to Plato. Against, see de Vogel, op. cit. For a different approach, see A. E. Taylor, *Plato, The Man and His Work* (New York 1956) 514.

[52] Alex. apud Simpl. *Phys.* 455 ll. 8–9, ad Arist. 202 b 36: ἔλεγε δὲ καὶ τὰς ἰδέας ὁ Πλάτων ἀριθμούς. εἰκότως ἄρα τὰς ἀρχὰς τοῦ ἀριθμοῦ καὶ τῶν ἰδεῶν ἀρχὰς ἐποίει.

[53] Aristot. Frag. 9 Rose: ὥστε εἰ ἄλλος ἀριθμὸς αἱ ἰδέαι, μὴ μαθηματικὸς δέ, οὐδεμίαν περὶ αὐτοῦ σύνεσιν ἔχοιμεν ἄν· τίς γὰρ τῶν γε πλείστων ἡμῶν συνίησιν ἄλλον ἀριθμόν;

[54] Wilpert, op. cit. 161–162. 168. A summary of the literature on this problem is brought by Gaiser, op. cit. 363–364 n. 92.

such it mediates between the one and infinity[55]. So far, this means reduction of the idea to number. Actually, the structure, the physis of the idea is number. The idea participates in number in the sense that every number can be said of more than one idea. The ontological significance of this is that number is a substance prior to the idea, as ἓν ἐπὶ πολλῶν, in the same way as the idea is a substance prior to sensuous things. But, unlike the relation between the ideas and the sensuous things, between numbers and ideas there is no *chorismos*: «Aber einem Chorismos der Zahlen von den Ideen, ähnlich dem der Ideen von den Dingen, stehen doch verschiedene Hemmnisse entgegen. Vor allem ist das Formalprinzip des ἓν, das gleichzeitig die erste Zahl ist, zugleich auch die erste Idee und hat diesen Platz schon lange inne. Von dieser Spitze aus entfalten sich die anderen Zahlen nach dem Verhältnis von Gattung und Art, genau so wie sich die Ideen entfalten. Diese Entwicklung der einzelnen Zahlen aus den höheren ist gleichzeitig die Entstehung der dieser jeweiligen Zahl entsprechenden Ideen. Mit der Zahl sind die ihr entsprechenden Ideen gegeben. Der Umstand, dass die Bestimmungsglieder der Ideen massgebend sind für ihr Zahlverhältnis, führt dazu, eine ebensolche Pyramide der Zahlen zu schaffen wie sie die Ideen bilden, und die Gleichheit dieser beiden Diairesispyramiden muss wiederum einer Trennung von Zahlen und Ideen hemmend im Wege stehen. So sind die Idealzahlen nicht getrennt von den Ideen, die sie bezeichnen, sondern sind nur in diesen Ideen. So eigenartig das klingt, Platon scheint in diesem Verhältnis von Idee und Zahl die aristotelische Lösung des Universalienproblems vorweggenommen zu haben. Die Zahl vier umfasst die Ideen des Körpers, der Gerechtigkeit, der Meinung, aber sie ist nur als Idee des Körpers, der Gerechtigkeit, der Meinung. Nirgends wird uns berichtet, dass Platon zunächst aus den Prinzipien des ἓν und der ἀόριστος δυάς die Zahlen abgeleitet habe und dann aus diesen die Ideen. Vielmehr ist mit der Idealzahl Zwei sofort auch die Idee der Linie gegeben, die 'an der Zwei teilhat'. Die Ideen sind nichts anderes als Zahlen, mit der Aufklärung dieses ihres Zahlcharakters, der 'Zurückführung auf Zahlen', haben sie selbst ihre Begreiflichkeit wiedergewonnen»[56].

It seems to me that in this particular point Wilpert extracts from his sources more than there is in them, and keeps not in line with his main argument:

1. In the above exposition, the relation of the idea of the body, the idea of justice and the idea of opinion to the number 'four' is exactly – as Robin would

---

[55] Cf. Ross, Introduction to *Aristotle's Metaphysics* I, lxviii: «Aristotle's way of putting the matter, that for Plato 'the Ideas are numbers', suggests that the numbers were not for Plato (as Zeller thought) mere symbols of the Ideas, but rather the last product of the abstractive process which had originally led him from the sensibles to the Ideas. In describing the Ideas as numbers, as successive products of the One and the great-and-the small, he may have seemed to himself to be stating in the clearest way the fact which is so often expressed in the later dialogues, that in the ideal world itself there is multiplicity as well as unity.»

[56] Op. cit. 170–171. As to the «Ableitung» of the numbers from the ἓν and the ἀόριστος δυάς and the ideas from the numbers, I think Theophr. *Met.* 6 b 11–14 RF (quoted above, p. 87) could provide a counter-example.

have it – as the relation of the sensuous particulars to the idea. The number is ἓν ἐπὶ πολλῶν, and these ideas are, according to Wilpert, specifications of the number 'four'. The idea of the body or the idea of justice have in them more determinations than has the number 'four' precisely because they participate in it. On Wilpert's own premises, the idea of the body cannot be thought without the number 'four', but the number 'four' can be thought without the idea of the body, the idea of justice or the idea of opinion, severally or collectively.

2. Furthermore, the ideas exist in their rational transparency in that they are numbers, i.e., they cannot exist as rational ideas without their numerical («zahl-mässige») determinations. This is again equivalent to saying that number is a *condition* of the ideas. But, as we have seen, this is the logico-ontological sense of 'prior'. If therefore Wilpert's analysis is correct, it should lead only the more conclusively to the possibility that ideas were not identified by Plato with numbers.

3. But «eigenartig» indeed is his suggestion that Plato anticipated Aristotle's conception of *universalia in re*. If indeed numbers are *universalia* in regard of the ideas – and that they are so is apparent from the example of the number four and its instantiations –, then there would be no hindrance that this solution of the problem of the chorismos between numbers and ideas would not hold good also for the chorismos between the ideas and the sensuous things. In effect, the dia-lectical problem of the one and the many turns out to be the same problem in the sensuous world and in the world of the ideas (see above p. 81 and n. 28). So that had Plato solved this problem in such a manner in the world of ideas, he would have solved it also in the sensuous world. But had Plato anticipated Aristotle's claim of *universalia in re*, then the whole of Aristotle's argumentation against Plato and the Platonists in the Metaphysics and elsewhere would be entirely pointless!

The alternative is to have the μέθεξις of the ideas in the numbers as a different sort of μέθεξις from that of the sensuous particulars in the ideas. This differentiation is prima facie unwarranted and seems inorganic within the framework of the development of Plato's thought. As it stands, Wilpert's solution has all the ear-marks of a tour de force.

3. *The definition of number*

Notwithstanding these reservations, part of Wilpert's interpretation can, as it seems to me, be accepted, within a broader conception of number in general, and of ideal number in particular. Aristotle has two definitions of number:

a) Met. vii 13, 1039 a 12: ὁ ἀριθμὸς σύνθεσις μονάδων. x 1, 1053 a 30: ὁ δ' ἀριθμὸς πλῆθος μονάδων. Cf. x 6, 1057 a 3; xiii 9, 1085 b 22; xiv 1, 1088 a 5; Phys. iii 6, 207 b 7. This is also the Euclidean standard definition: Eucl. Elem. vii def. 2: ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος.

b) Met. v 13, 1020 a 13: $\pi\lambda\tilde{\eta}\theta o\varsigma$ τὸ πεπερασμένον ἀριθμός.

This second definition, as Ross remarks in his commentary to this passage, refers to the Academy, and more specifically to Eudoxus: «The definition of number as πλῆθος πεπερασμένον is anticipated by Eudoxos' definition of it as πλῆθος ὡρισμένον (Iambl. *in Nicom. Ar. Introd.* 10.17). ... Mr. F. M. Cornford (*Class. Quart.* xvii. 8n.) suggests (rightly, I think) that the present definition 'goes back to the characteristically Pythagorean conception of number as the product of the union of πέρας and ἄπειρον'; whereas such definitions as σύνθεσις μονάδων ..., πλῆθος μονάδων ... represent 'the crude, and so to say materialistic, view which may well have been shared by the Egyptians and the Pythagorean mathematicians or number-atomists' of the sixth century»[57].

But Ross does not point out the Platonic character of this definition. Wilpert too draws on Cornford's paper, but he has an important addition: «Ihr [der Definition der Zahl als πλῆθος πεπερασμένον] entspricht genau die platonische Zahlengenesis aus ἕν und ἀόριστος δυάς. Jedenfalls macht sich Platon diese Definition und Zahlenableitung zu eigen. Die Frage der Priorität zwischen Platon und den Pythagoreern muss dagegen einstweilen noch offen bleiben. Für Platon kann man die Eudoxische Definition direkt herauslesen aus dem Bericht Alexanders aus Περὶ τἀγαθοῦ bei Simpl. *Phys.* 455, 6–7 Diels»[58].

Book v of the Metaphysics is now unanimously regarded as an early book[59], and the concept of number found there is eccentric in relation to the main stream of Greek and Mediaeval philosophy and mathematics, as they developed mainly under the influence of Aristotle himself. As against the accepted concept of number as 'a plurality of unities', this other concept of number as 'determined plurality' is an eminently Pythagorean-Platonic concept, as can be clearly seen on comparison with the concept of number in, say, the Philebus. This concept too is found in Euclid, especially in Book v.

Whereas the first concept of number sees it essentially as *quantity*, the Platonic concept defines number essentially as *relation*. As Toeplitz[60] points out, the Greek – he should have better said: the Platonic – concept of number is first of all the concept of a relation between two magnitudes, and not primarily the concept of the cardinal number of a set. Quantity is for Plato a further determination of the category of relation. So, the great-and-small, as the prototype of all numerical

---

[57] Ross, *Aristotle's Metaphysics* vol. I 323–324.

[58] Op. cit. 177–178 n. 9. The passage in Simplicius is: καὶ γὰρ ἕκαστος τῶν ἀριθμῶν καθόσον μὲν ὁδὲ τίς ἐστι καὶ εἷς καὶ ὡρισμένος, τοῦ ἑνὸς μετέχει, καθόσον δὲ διαιρεῖται καὶ πλῆθός ἐστι, τῆς ἀορίστου δυάδος.

[59] Cf. e.g. Ross, Intr. to *Aristotle's Metaphysics* I, xxv; W. Jaeger, *Aristotle*, tr. R. Robinson (2nd ed., Oxford 1948) 169.

[60] Op. cit. (see above n. 34) 9: «Nicht etwa die μεγέθη, die allgemeinen Grössen von Euklid v, sind das griechische Substrat des modernen Zahlbegriffs, sondern die λόγοι, die Verhältnisse von zwei gleichartigen μεγέθη.» See also Klein, op. cit. (see above n. 11) 97 and n. 106, and p. 158.

determination, falls under the category of relation, and Plato does not recognize quantity as a separate category, alongside with relation[61].

Plato's approach to the problem of irrationals can be readily understood from his conception of number as relation. Plato held irrationals, such as $\sqrt{2}$, $\sqrt{3}$, etc., to be numbers (as opposed to Aristotle's view on this point[62]), because they express a relation (even if it be a relation that cannot be exhaustively expressed in a finite number of steps). And the Epinomis says[63] that geometry is the science of τῶν οὐκ ὄντων ὁμοίων ἀλλήλοις φύσει ἀριθμῶν ὁμοίωσις πρὸς τὴν τῶν ἐπιπέδων μοῖραν γεγονυῖα, whereas stereomety studies τοὺς τρὶς ηὐξημένους καὶ τῇ στερεᾷ φύσει ὁμοίους· τοὺς δὲ ἀνομοίους αὖ γεγονότας ἑτέρα τέχνῃ ὁμοιοῖ[64].

I shall not here moot the question whether Plato intended something like arithmetization of mathematics[65]. Anyway, from what is said above it is clear that Plato's concept of number was much broader than the merely quantitative, 'cardinal' concept. It is against this concept of number that Aristotle's criticism in Metaphysics xiii is directed (Met. xiii 2, 1077 a 9ff.): ἔτι γράφεται ἔνια καθόλου ὑπὸ τῶν μαθηματικῶν παρὰ ταύτας τὰς οὐσίας. ἔσται οὖν καὶ αὕτη τις ἄλλη οὐσία μεταξὺ κεχωρισμένη τῶν τ'ἰδεῶν καὶ τῶν μεταξύ, ἢ οὔτε ἀριθμός ἐστιν οὔτε στιγμαὶ οὔτε μέγεθος οὔτε χρόνος. εἰ δὲ τοῦτο ἀδύνατον, δῆλον ὅτι κἀκεῖνα ἀδύνατον εἶναι κεχωρισμένα τῶν αἰσθητῶν. Cf. Ross, ad loc. (II 413): «The general mathematics here referred to, which proves attributes that are not peculiar to numbers or to spatial magnitudes or to times, is also mentioned in 1077 b 7, E. 1026 a 27, An. Post. 74 a 23. Eudoxos' doctrine of proportion, which is preserved in Euclid's Elements, Bk. V, is the best instance of this 'general mathematics'»[66].

4. Ideal numbers

Number, being situated between the irrational One and the irrational infinity, is the only guarantee of rationality. As 'determined plurality', number is articulated: plurality becomes measured and determined by the one-limit. Number insofar as it is articulated, has genesis: it 'becomes' out of the one and the duality (or out of limit and the unlimited), and its rationality consists in the possibility of describing its becoming out of its elements.

---

[61] Cf. Wilpert, op. cit. 109–110.

[62] Cf. his views on the essential (τῷ γένει) difference between arithmetic and geometry in Anal. Post. i 7; cf. further Proclus In primum Euclidis Elementorum librum p. 60 l. 7 Friedlein.

[63] Of course a proof of authenticity is impossible. On the other hand, even those who are sceptical about the authorship of the Epinomis agree that it is Platonic in character.

[64] 990 d 2–9; cf. Meno 82 b ff., Theaet. 147 d ff.

[65] On the whole of this problem see A. E. Taylor, Form and Number, Mind 36 (1927) 426–427, Recens. of Stenzel, Zahl u. Gestalt, Gnomon 2 (1926) 396ff.; Toeplitz, op. cit.; Stenzel, op. cit., Nachtrag 184–185; R. Lacey, The mathematical passage in the Epinomis, Phronesis 1 (1956) 81–104.

[66] Cf. Toeplitz, op. cit. (see above n. 34) 28 n. 25: «Im Zentrum der Beschreibung der Mathematik steht unzweideutig die allgemeine Proportionenlehre (1077 a 9).» Cf. further ibid. 28.

The rational system of numbers constitutes the prototype and skeleton of the rational system of ideas. Stenzel has shown the parallelism between the derivation of numbers and that of the ideas: the diairesis of numbers and the diairesis of ideas have the same structure. And just as numbers are derived according to an order of prior and posterior, so are the ideas derived according to such an order[67].

The primordial character of number in respect of its rationality is not its *quantity*, but *its being hierarchically related to other numbers*. Also the structure of the idea is determined by its relations to other ideas, even as the structure of number is determined by its relations to other numbers, and, in last analysis, by the special relation within it of the one and the indeterminate duality. From the sole point of view of its rationality, we can abstract from the quantitative ('cardinal') aspect of number and look upon it as a sheer system of prior and posterior. The broader Platonic concept of number, as it is introduced above, affords us to go beyond the 'cardinal' aspect of number. Arithmetic, geometry and even dialectic become now different instantiations of a single rational system, as Stenzel has shown at length.

The idea is number insofar as it is determined plurality, according to an order of prior and posterior. The relations that hold between the ideas are numerical («zahlmässige») relations. The diairesis gives us the sequence of conditions and conditioned, and with it, in every particular idea, its components-conditions. But the diairesis, although it is ordered numerically, does not necessarily deal with quantities. In this respect one could perhaps accept Natorp's view on Plato's envisaged «algebra without concepts of quantity»[68].

Every idea is determined by the diairetic-mathematical procedure, which allots it its 'place' among other ideas and grants it its individuality by way of exact determination of its components. Just as number secures a foothold between the one and infinity, in like manner the idea, completely defined by its determinations, secures a standing point of reference between the alogon and the absolute flow. It is not only a question of a fixed number of steps in the diairetic procedure (i.e., of fixed number of determinations in the idea). There is more to it: there are in the diairetic procedure fixed relations between these determinations, a structure that discloses itself in the dialectical derivation and appears as a real constituent of the idea itself. This diairetic structure, previous to its being a matter of number of steps, is a matter of relations of 'prior' and 'posterior', of the articulation of a plurality which is not necessarily – as in the diairesis of the ideas – a plurality of quantities, i.e. a plurality of homogenous quantities. Plato's contention is that there is lawfulness in the derivation of qualitative concepts, just as there is such

---

[67] Cf. Arist. *Met.* iii 3, 999 a 8; xiii 6, 1080 b 11. Trendelenburg connects these places with *Met.* v 11, 1019 a 1 (quoted by Ross ad loc.). Cf. further Stenzel, op. cit., ch. III. For a survey of the different suggestions for the derivation of numbers in Plato, see Wilpert, op. cit. 202ff.

[68] Op. cit. 418–419. Cf. Ritter, op. cit. 178; Klein, op. cit. (see above n. 11) 92.

a lawfulness among numbers[69]. From this point of view, it seems to me, it can be said that the ideas are numbers.

It seems, thus, that *ideal number is number insofar as it represents rationality.* The question of how many are the ideal numbers seems – from this point of view – superfluous[70].

5. Relation and substance

The subordination of the ideas to number brings the ideas, as Alexander rightly points out, under the category of relation (or, at least, under the category of quantity, which is not much better)[71]. This priority given to the category of relation over the category of substance follows necessarily from the above interpretation of the theory of ideal numbers, but implies serious difficulties.

This objection against the nature of the Platonic idea was raised also with no direct connection to its numerical character. Other considerations, though not completely unrelated to these, lead also to the priority of the category of relation over the category of substance – at least in regard to some aspects of the idea[72].

One line of Aristotle's criticism of Plato in Metaphysics i proceeds from the character of the idea as model. All that exists in the sensuous world is but a copy of the ideal model. On the other hand the nature of the idea is to be such a model. But in this Plato (Wilpert says: the Academy) makes the idea itself something relational, insofar as 'model' is a correlative concept: there is no model without a copy. This relation of model and copy is essential to the idea: it cannot be model – and this is its essence – if a copy does not exist as its counterpart. And so, even by considerations drawn from the theory of ideas itself, the ideas are dependent on their correlates. The theory of ideas – Wilpert sums up, following Aristotle – gives priority to the category of relation over the category of substance[73].

The relationality of the idea is emphasized in the interpretation of the ideas as functions of knowledge. In this interpretation, the idea is seen in its role *within* knowledge alone, and, therefore in its relatedness, or – in Natorp's term – in its functionality. The critical presentation of the problem, which posits knowledge as the primary fact, takes, ipso facto, as its starting point, the category of relation.

But the idea has another side as well, and Plato would not give it up, though he would not prove it satisfactorily: its substantial side. The ideas are functions

---

[69] Cf. Natorp, op. cit. (see above n. 11) 419–420. 433; E. Zeller, *Plato and the Older Academy,* tr. S. F. Alleyne and A. Goodwin (New York 1888) 256.

[70] Cf. Stenzel, op. cit. 173; C. J. de Vogel, *Pythagoras and Early Pythagoreanism* (Assen 1966) 202.

[71] See above p. 92s. and n. 61.

[72] The distinction between the category of substance and the category of relation appears already in the dialogues. See *Soph.* 255 c 12: Ἀλλ'οἶμαί σε συγχωρεῖν τῶν ὄντων τὰ μὲν καθ' αὑτά, τὰ δὲ πρὸς ἄλλα ἀεὶ λέγεσθαι. Cf. *Charm.* 168 b ff., *Rep.* 438 a, *Theaet.* 106 b, *Parm.* 153 c, *Phil.* 51 c.

[73] Op. cit. 111–112.

of knowledge, but they are as well substances existing *καθ'αὐτά*. Renouncing this essential duality of the idea would lead, on the one hand, to a strict chorismos as with Parmenides, or, on the other hand, to the absorption of the ideas into the soul and with this to the abolition of all valid knowledge. This can be clearly seen in Haezrahi's extensional interpretation of the *παντελῶς ὄν*, in which it is shown that the ideas cannot be the *παντελῶς ὄν*, because they stand in relation to the sensuous world, and the soul. The extensional interpretation of the *παντελῶς ὄν* postulates the duality of the idea, in that it requires the separate existence of ideas, souls and the sensuous world, and points to the functional relations that lie at the basis of the general system.

The paradox in the categorical status of the idea is especially apparent in what is called by Haezrahi «the secondary or the sixth mode of existence» of the genera of Being: namely «the existence in the soul of all the genera of Being that we have reckoned – as objects of its knowledge, as objects of its desire, and as the crystallization of its discoveries about them (in the sciences, and in philosophy, in poetry and in the arts)». «The sixth mode of being ... is realized ... as the image of the primary mode in the psychic-epistemological material, and as the expression of this image, mainly in words, but also in the different arts, as expression and as existence in the corporeal world. The sixth mode of being is a secondary mode of being in the soul, or in the corporeal world, and it has no reality apart from these two presentations. 'The perfect being' ought to allow the realization of the sixth mode of being as secondary, image-like being ... without, for this reason, constituting a separate mode of being in itself»[74].
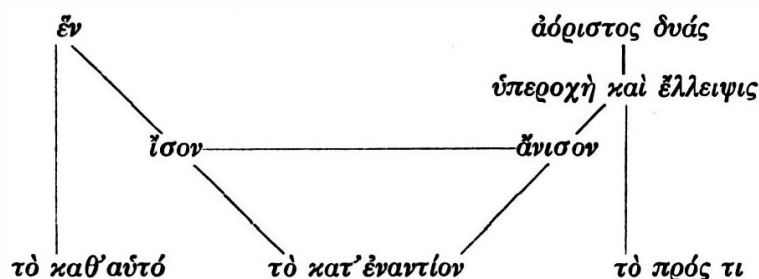
Therefore, in the idea, the substance is prior to the relation, inasmuch as the idea is a separate idea, a *real* object of knowledge, existing *καθ'αὐτό*, which ensures by its full existence the certainty of knowledge. Whereas, insofar as the idea is an epistemological function existing secondarily in our knowledge, relation is prior in it to substance. Concerning its nature, *τῇ φύσει*, this mode of being is secondary, but, *πρὸς ἡμᾶς*, in what concerns the grounds for believing in the existence of ideas, and especially in what concerns the proofs *ἐξ ἐπιστημῶν* and *κατὰ ἓν ἐπὶ πολλῶν*, the idea is first and foremost a function, a relate.

The argument *κατὰ ἓν ἐπὶ πολλῶν* brought us to a view of the idea as a structured unity, as 'a determined plurality', as number composed out of limit and the unlimited, and as such, under the category of the *πρός τι*. But what was disclosed to us at the end was already implicit in the starting point: If the arguments for the existence of ideas start from the sensuous world – and such are Plato's arguments, even if they aim at 'what eternally is' – if such are his arguments, then in this very first stage, by the very formulation of the question of the one and the many, the idea was already made into something relational. Nevertheless, at the same time, and not the less vigorously, Plato demands the substantiality of the

---

[74] Haezrahi, op. cit. (see above n. 9) 339.

idea. And as with the idea, so with number: number is at one and the same time both relation and substance[75].

---

[75] It is interesting to compare with this conclusion the reconstruction by Wilpert, op. cit. 191, of the Platonic system of categories, after Sextus Empiricus: «Das Schema der platonischen Reduktionen ist folgendermassen darzustellen:

$$\begin{array}{cc}
\text{ἕν} & \text{ἀόριστος δυάς} \\
& \text{ὑπεροχὴ καὶ ἔλλειψις} \\
\text{ἴσον} \longrightarrow \text{ἄνισον} & \\
\text{τὸ καθ'αὑτό} \quad \text{τὸ κατ'ἐναντίον} & \text{τὸ πρός τι}
\end{array}$$

Beherrscht ist diese Prinzipienableitung von dem Gedanken, die schon in der Einleitung des Seienden zutage tretende Scheidung in Absolutes und Relatives als durchgehendes, alles Seiende beherrschendes Strukturgesetz zu erweisen. Am Ende dieser Reihe kann nichts anderes stehen als die absolute Bestimmtheit, verkörpert durch das ἕν, und die reine Relativität und Unbestimmtheit der ἀόριστος δυάς.» – For another scheme of categories, see Gaiser, op. cit. 77 and passim.